SPECIAL PROJECT FINAL REPORT

All the following mandatory information needs to be provided.

Project Title:	**Direct numerical simulation of long-term evolution of wind waves: dynamics vs kinetics, with applications to freak waves prediction**
Computer Project Account:	**spgbshri**
Start Year - End Year :	**2019 – 2021**
Principal Investigator(s)	**Prof V.I. Shrira**
Affiliation/Address:	**School of Computing and Mathematics, Keele University, Keele ST5 5BG UK**
Other Researchers (Name/Affiliation):	**Dr S. Annenkov, School of Computing and Mathematics, Keele University, Keele ST5 5BG UK**

The following should cover the entire project duration.

Summary of project objectives

(10 lines max)

The target of the project is to perform long term DNS simulations and to compare them with high quality observations. As shown by Annenkov & Shrira (2014), higher-order moments of a random wave field, and hence the probability of freak waves, are dependent on spectral shape, not just on integral characteristics of a wave field. Preliminary estimates show that the found discrepancies in spectral shape can strongly affect higher-order moments, in particular kurtosis, and the difference is expected to be substantial (of the order of 100%). This has huge potential implications for the prediction of extreme wave events. Specific objectives include exploring the discrepancy between the shape of the DNS (verified by observations) and the Hasselmann equation predictions, examining implications for probability of freak waves, mixing via the vortex force and other processes sensitive to the shape of spectra, getting new insights into the input and dissipation functions.

Summary of problems encountered

(If you encountered any problems of a more technical nature, please describe them here.)

No particular problems encountered

Experience with the Special Project framework

(Please let us know about your experience with administrative aspects like the application procedure, progress reporting etc.)

More than 10 years of continuous experience with all aspects of special projects application procedures and reporting

Summary of results

(This section should comprise up to 10 pages, reflecting the complexity and duration of the project, and can be replaced by a short summary plus an existing scientific report on the project.)

In this project, we study numerically the long-term evolution of water wave spectra, using the standard model based on the Hasselmann kinetic equation (KE), widely used in operational wind wave forecasting, as well as two other models developed by us and based upon different sets of assumptions. The second model is the generalised kinetic equation (gKE), using the same statistical closure as the KE, but derived without the assumption of quasi-stationarity. The gKE does not employ the quasi-stationarity assumption and is valid when a wave spectrum is changing rapidly (e.g. at the initial stage of evolution of a narrow spectrum, or after a rapid change of wind forcing). The third model is based on the Zakharov integrodifferential equation for water waves and does not depend on any statistical assumptions. Since the Zakharov equation plays the role of the primitive equation of the theory of wave turbulence, we refer to this model as direct numerical simulation of spectral evolution (DNS-ZE). The DNS-ZE method allows to study long-term spectral evolution (up to $O(10^4)$ periods), which was previously possible only with the KE. Thus, we are able to perform a direct comparison of spectral evolution with and without the statistical closure.

During the previous Special Project, which ended in 2018, we traced the evolution of initially narrow (both in frequency and angle) spectra subjected only to dissipation at high frequencies, and considered the evolution of higher statistical moments. In the current Special project, the target is to study the evolution of waves generated by wind blowing in the offshore direction. In this report, we present numerical results, obtained with three different models, on spectral evolution of wind waves generated by a moderate or strong wind in an initially quiescent sea. We compare the evolution of various spectral parameters, including spectral peak parameters, angular width, and higher moments, emphasizing differences between the models. Then we compare the results obtained with all three models with airborne measurements collected in two different experiments: GOTEX experiment off the coast of Mexico in February 2004, and HyMeX experiment in the western Mediterranean sea in February and March 2013.

I. Wave action spectra and spectral peak

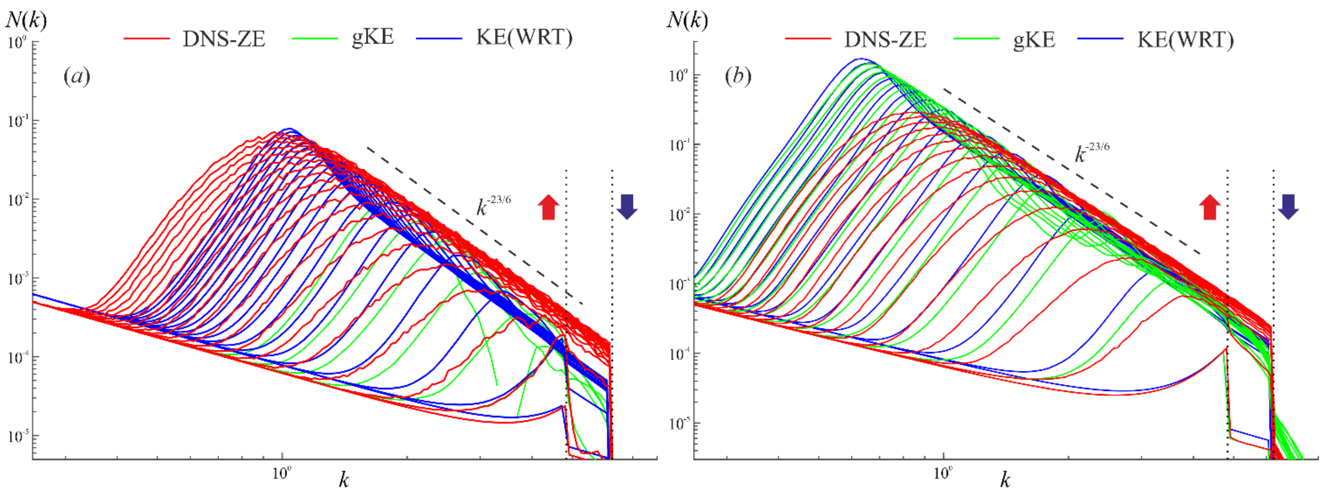

Figure 1. Long-term evolution of wave action spectrum generated by constant wind over initially quiescent sea, with a direct comparison of three different numerical methods. Evolution is simulated by the DNS based on the Zakharov equation (DNS-ZE), standard kinetic equation, WRT algorithm, (KE(WRT)), and generalized kinetic equation (gKE). (*a*) Wind speed $\frac{U}{c_0} = 1.2$, where c_0 is the characteristic phase speed, spectra are plotted every 1000characteristic periods (b) Wind speed $\frac{U}{c_0} = 2.0$, spectra are plotted every 320 characteristic periods

Figure 1 shows the development of wind wave spectra generated by constant wind with speed U equal to $1.2c_0$ and $2.0c_0$, where c_0 is the characteristic phase speed, approximately corresponding to phase speed at the end of evolution. These values of wind speed are chosen since they are close to minimal and maximal wind speeds in the KuROS experiment. Wind forcing is calculated according to Hsiao & Shemdin (1983), dissipation is applied to small scales.

Spectral evolution demonstrated by all three models in both cases shown in figure 1 is clearly self-similar, with spectral slope corresponding to the theoretical $k^{-23/6}$ angle. Both kinetic equations give nearly identical results, although the gKE develops a certain instability in smaller scales at large time, especially manifested for weaker wind, where the simulation with the gKE even has to be abandoned prematurely. The downshift due to kinetic models is slightly faster for stronger wind, and the spectral peak is more pronounced. The most striking difference between the kinetic models and the DNS is in the shape of the spectral peak, which is narrower with the kinetic models, and the front appears to be much steeper.

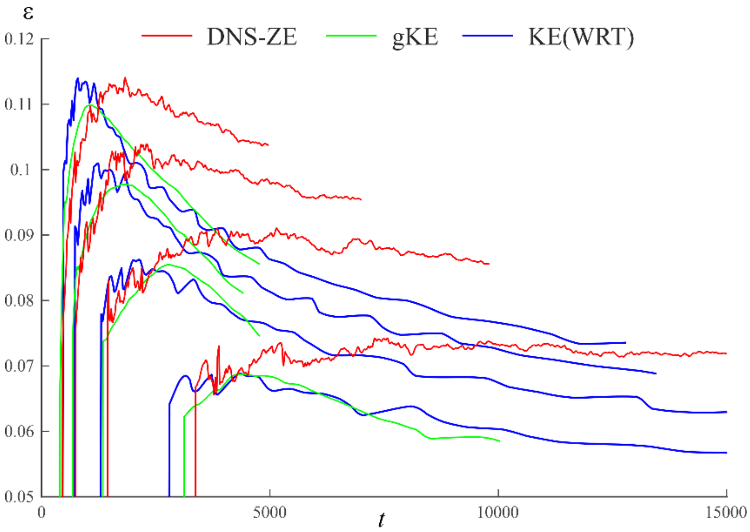

Figure 2. Evolution of wave steepness with time (in characteristic periods) in a developing wave action spectrum, generated by constant wind U/c_0 = 1.2, 1.5, 1.8 and 2.1, simulated with three numerical methods

Evolution of wave steepness for four different wind speeds is shown in figure 2. After the initial rise to a maximum, wave steepness slowly decreases in all models, although the decrease is faster for the kinetic equations, especially for stronger winds.

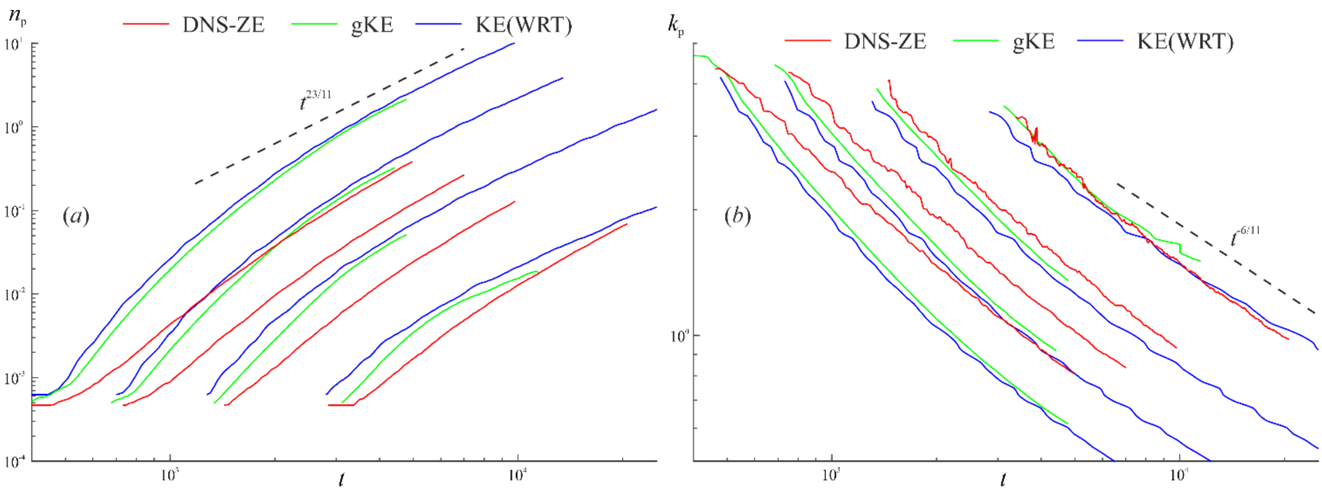

Figure 3. Evolution of spectral peak with time (in characteristic periods) in a developing wave action spectrum, generated by constant wind U/c_0 = 1.2, 1.5, 1.8 and 2.1, simulated with three numerical methods. (a) spectral peak amplitude (b) peak wavenumber

In figure 3, evolution of spectral peak parameters (amplitude and position of the peak) is shown. In all cases, for large times the values approach theoretical asymptotes known from the analysis of the Hasselmann kinetic equation (Badulin et al 2005). At the same time, for stronger wind there is a large discrepancy in peak amplitude between the kinetic equations and the DNS, which also appears to be growing with time. For weaker wind, this effect almost disappears.

However, for all wind speeds there is a considerable difference is spectral shape: DNS spectra appear to be wider, although a more detailed analysis shows that the width parameters are in fact close, the difference in spectral shape is being manifested by lower *peakedndess* of DNS spectra, rather than different width. This difference in peakedness is best represented by the peakedness parameter

$$\Phi = \frac{\int_0^\infty E(\omega)^2 \mathrm{d}\omega}{\left[\int_0^\infty E(\omega)\mathrm{d}\omega\right]^2},$$

where $E(\omega)$ is the energy-frequency spectrum (Goda 1970).

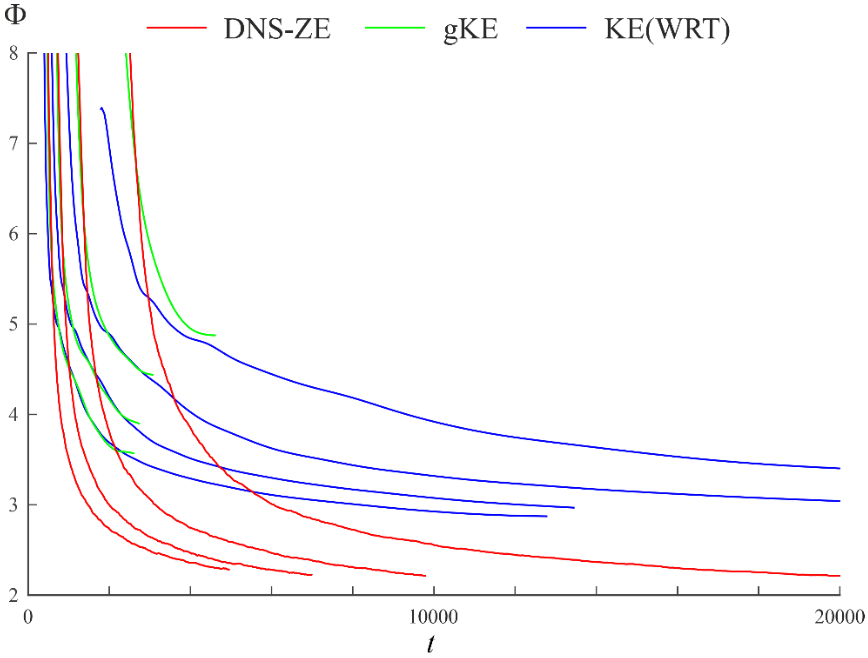

Figure 4. Evolution of spectral peakedness Φ with time (in characteristic periods) generated by constant wind U/c_0 = 1.2, 1.5, 1.8 and 2.1, simulated with three numerical methods

The evolution of the peakedness parameter Φ is shown in figure 4. Peakedness for all DNS simulations is much lower than for the corresponding simulations with the kinetic equations, demonstrating the difference in spectral shapes.

II. Angular width

Here we consider the evolution of mean directional spread

$$\theta_m = \overline{\theta_2(k)},$$

defined as the average of the second moment of $D(\theta)$

$$\theta_2(k) = \left(\int_0^{\pi/2} \theta^2 D(k,\theta)\,\mathrm{d}\theta\right)^{1/2} \left(\int_0^{\pi/2} D(k,\theta)\,\mathrm{d}\theta\right)^{-1/2},$$

where $D(k,\theta)$ is the angular distribution function (Hwang *et al* 2000). Unlike in the absence of wind forcing (cf the final report of the previous Special Project, and also Annenkov & Shrira 2018), the evolution of mean directional spread for wind waves, shown in figure 5, is close for the DNS and the kinetic equations. However, it is clear that due to the integral nature of the averaged directional spread, it is mostly defined by the spectral tail, while for practical applications the angular width of the spectral peak is important (Badulin & Zakharov 2017).

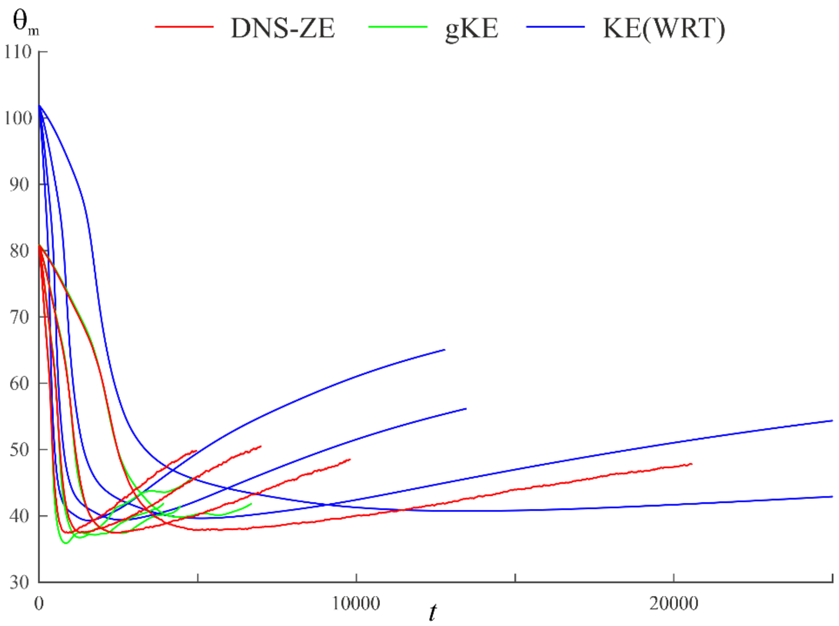

Figure 5. Evolution of average angular width θ_m with time (in characteristic periods) in a developing wave action spectrum, generated by constant wind U/c_0 = 1.2, 1.5, 1.8 and 2.1, simulated with three numerical methods

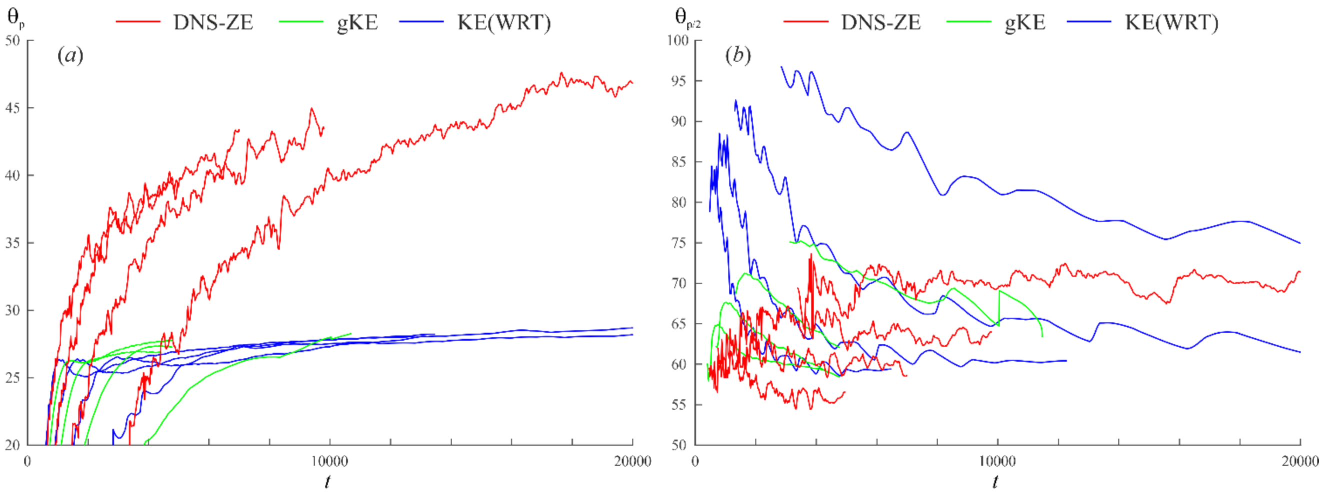

Figure 6. Evolution of angular width θ at (a) wavenumber of the spectral peak and (b) half of wavenumber of the spectral peak with time (in characteristic periods) in a developing wave action spectrum, generated by constant wind U/c_0 = 1.2, 1.5, 1.8 and 2.1, simulated with three numerical methods

In figure 6, evolution of the second moment of the directional distribution $\theta_2(k)$ is shown for $k = k_p$ and $k = 0.5k_p$. At these low wavenumbers, the evolution of angular width is markedly different between the kinetic equations and the DNS. At the spectral peak, while the kinetic equations show that the spectral peak is narrow in angle and nearly constant in time, the DNS demonstrates an increase of the angular width to a much higher value. At low frequencies, both the gKE and the DNS are quite different from the Hasselmann kinetic equation, which gives much larger angular width.

III. Higher statistical moments

In the context of gravity water waves, kurtosis is commonly used as the main characteristics of the field departure from gaussianity. The component of the kurtosis due to wave interactions (the dynamical kurtosis) is usually assumed to be small for a quasi-stationary wind wave field, which is confirmed by the current simulations. The other component, the bound harmonic kurtosis, can be calculated from the spectrum as

$$C_4^{(b)} = \frac{m_4^{(b)}}{m_2^2} - 3,$$

where

$$m_4^{(b)} = 3\int \omega_0\omega_1\, n_0 n_1 \mathrm{d}\boldsymbol{k}_{01} + 12\int \mathcal{J}_{012}^{(4)}\omega_0\omega_1\omega_2\, n_0 n_1 n_2 \mathrm{d}\boldsymbol{k}_{012},$$

$m_2 = \int \omega_0 n_0 \mathrm{d}\boldsymbol{k}_0$, $n_j = n(k_j)$, $\mathrm{d}\boldsymbol{k}_{01} = \mathrm{d}\boldsymbol{k}_0\mathrm{d}\boldsymbol{k}_1$, etc., and the coefficient $\mathcal{J}_{012}^{(4)}$ was derived by Janssen (2009) (see also Annenkov & Shrira 2013).

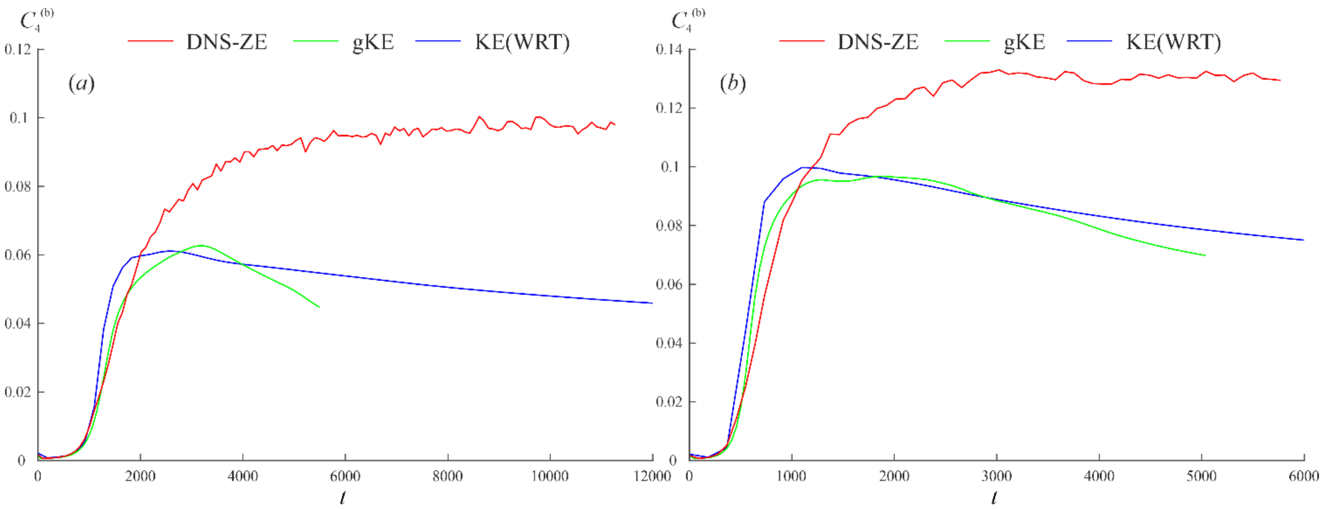

Figure 7. Evolution of bound harmonic kurtosis $C_4^{(b)}$ with time (in characteristic periods) in a developing wave action spectrum, generated by constant wind (*a*) U/c_0 = 1.5 (*b*) U/c_0 = 2.0

Evolution of $C_4^{(b)}$ for two wind speeds, obtained with different models, is shown in figure 7. The difference in spectral shapes manifests itself in much higher values of the bound harmonic kurtosis in the DNS simulations, corresponding to increased probability of extreme wave events.

IV. Comparison with field measurements.

In the final part of the project, we compared the results obtained with all three models with airborne measurements collected in two different experiments: GOTEX experiment off the coast of Mexico in February 2004 (Romero & Melville 2010), and HyMeX experiment in the western Mediterranean sea in February and March 2013 (le Merle et al 2019). In both cases, the wave field was generated by a strong quasi-stationary offshore wind jet, which is caused by pressure differences and accelerates passing through a valley into the sea. For modelling of waves off the Mexican coast, the initial conditions were taken from the measured spectrum at the moment when wind waves prevail over swell after a short initial part of the evolution. Waves in the Mediterranean Sea were modelled with constant wind forcing and zero initial condition.

The evolution of integral characteristics, e.g. significant wave height and wave steepness, is reproduced reasonably well by all modelling approaches. However, the spectral shape of developed waves demonstrates a large discrepancy between, on the one hand, the measured spectra and the DNS modelling and, on the other hand, spectra modelled by both kinetic equations (figure 8).

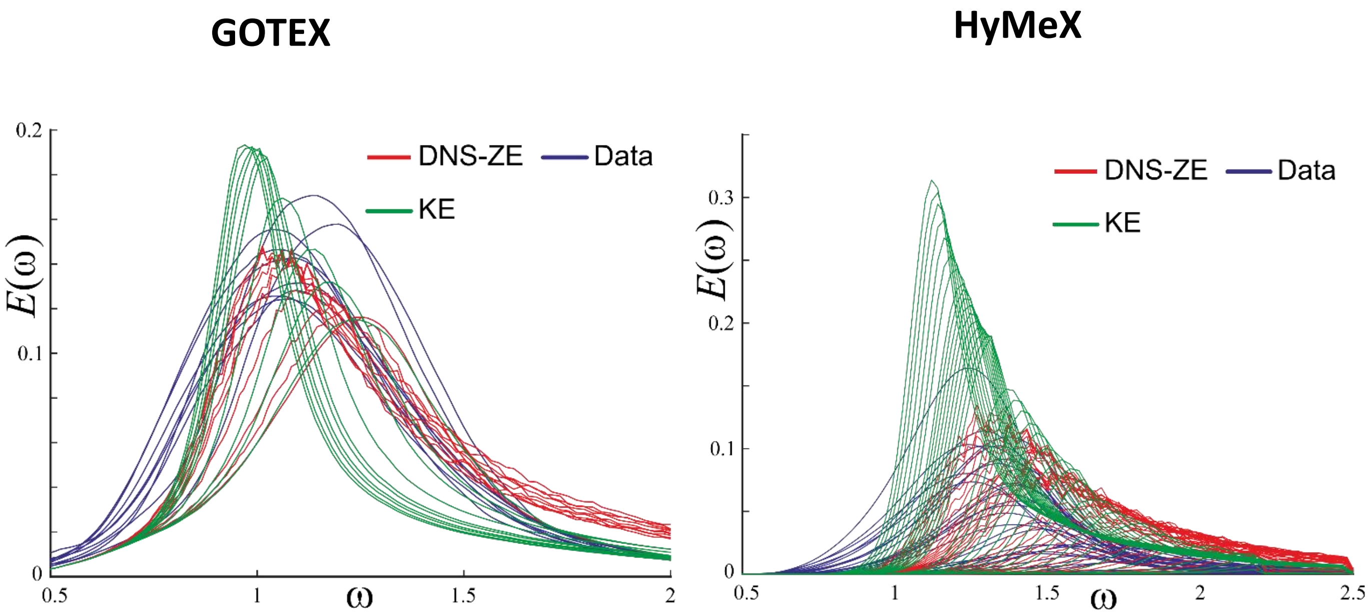

Figure 8. Spectral development in modelling of GOTEX and HyMeX data. Measured spectra (blue curves), DNS spectra (red curves), KE spectra (green curves)

At the intermediate and advanced stage of development, both measured spectra and the DNS spectra tend to Pierson-Moskowitz spectral shape, while the modelling based on the kinetic equations invariably predicts spectra with a higher, more pronounced peak. In terms of the parameter of spectral peakedness, a commonly convenient measure of spectral shape, a large (of order one) discrepancy was found (figure 9).

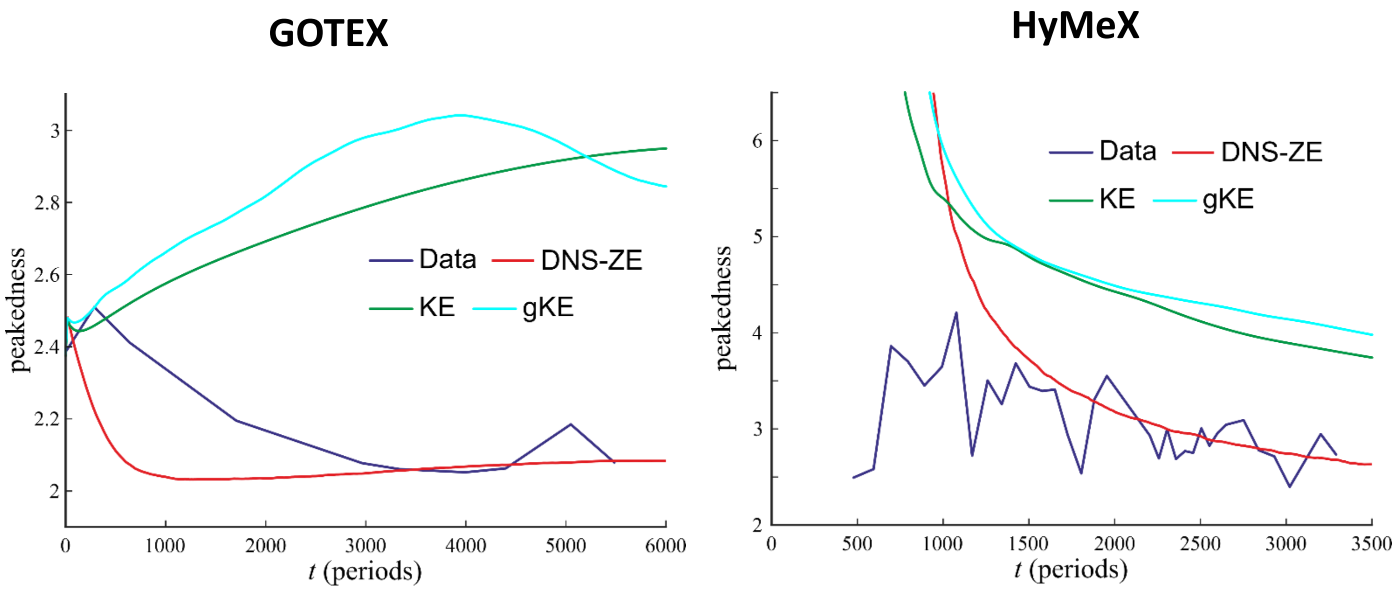

Figure 9. Spectral peakedness $Q_p = \frac{2\int \omega S(\omega)^2 d\omega}{[\int S(\omega) d\omega]^2}$ (Goda 1970) as function of time in GOTEX and HyMeX experiments, and in the simulations by DNS and the kinetic equations.

Thus, we have demonstrated that spectral shapes obtained with the DNS are in much closer agreement with measurements of mature sea states. This discrepancy between the spectral shapes of mature oceanic waves and solutions of the Hasselmann equation is long known as a major problem in wind wave modelling. For the first time, we have shown that it can be overcome with DNS, pointing to the nonlinear interaction term of the Hasselmann equation as the origin of the discrepancy. Although the discrepancy mostly concerns spectral shapes, rather than spectral integrals, it has huge significance for many applications, including freak wave prediction. In particular, it was shown that the demonstrated difference in spectral shape corresponds to an order one change of kurtosis, which dramatically affects the freak wave probability estimates. On the other hand, since the existing wind wave models are optimised against the available measurements, knowledge of systematic errors in models can drastically improve the quality of such optimisations, and thus improve the quality of predictions even for spectral integrals like significant wave height.

V. Theoretical explanation of the discrepancies, and the role of finite non-gaussianity

The origin of the discrepancy has been identified as the neglect of non-gaussianity in the derivation of the Hasselmann equation (Annenkov & Shrira 2022). While the Hasselmann equation takes into account finite nonlinearity, it assumes infinitesimal non-gaussianity, since in its derivation the six-point correlator is expressed in terms of two-point ones, neglecting the four-point cumulants in the expansion. Although this approximation, which takes into account only the leading term in the expansion of the correlator, is a prerequisite for obtaining the kinetic equation for spectral amplitudes in the closed form, it leads to the equation with the right-hand side being a homogeneous function of the spectrum, thus neglecting all effects of finite nonlinearity.

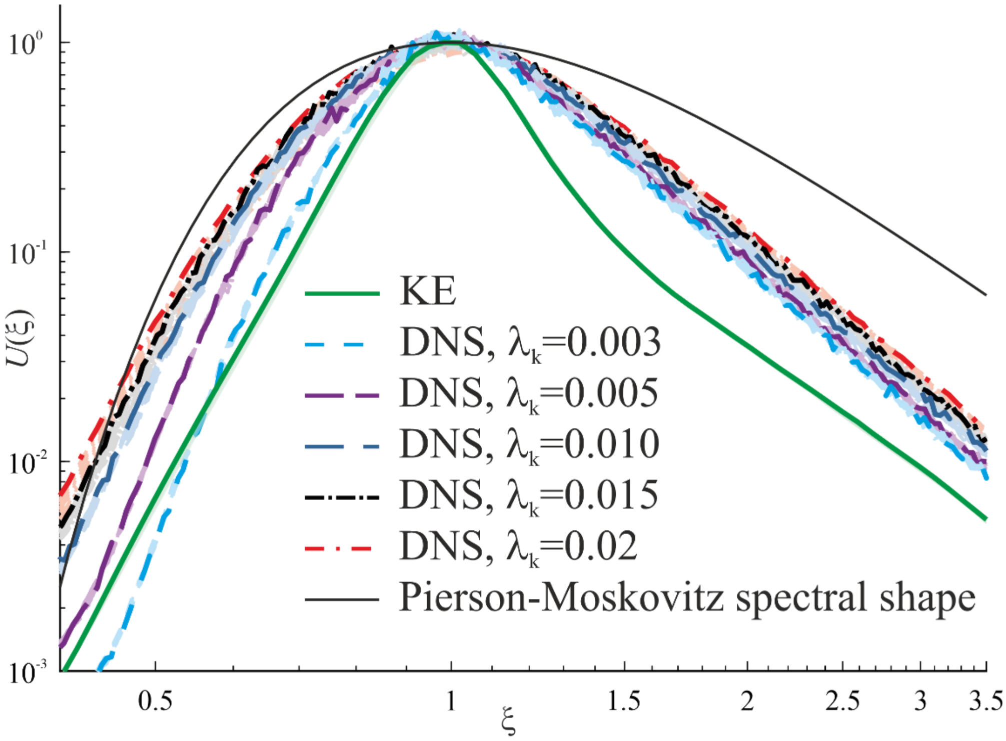

Figure 10. Self-similar shape function extracted from the numerical solutions at the last 1000 wave periods of evolution. Shapes at every 100 periods are shown in light colours, the final curve is in darker colour of the same hue. Numerical models used are the KE and the DNS on 161x71 (DNS161) and 321x71 (DNS321) grids for different values of λ_k.

We show by DNS that although non-gaussianity is weak, in the long term it leads to a considerable distortion of the spectral shape. Here, we are helped by the particular design of the DNS algorithm. The algorithm is based on the idea of coarse-graining of a wave field (Annenkov & Shrira 2018), which relaxes the resonance condition into $\boldsymbol{k}_0+\boldsymbol{k}_1-\boldsymbol{k}_2-\boldsymbol{k}_3=\Delta\boldsymbol{k}$, in contrast to the standard condition $\Delta\boldsymbol{k} = 0$. Here $|\Delta\boldsymbol{k}|/k_{min} < \lambda_k \overline{\omega}/\omega_{min}$, and the crucial role is played by parameter λ_k (the coarse-graining parameter). If $\lambda_k = 0$, there are no interactions (on a log-spaced grid), and a wave field is gaussian. When λ_k is increased from zero, the number of wave interactions grows quadratically, the rate of spectral evolution quickly increases and then saturates, on a certain value of λ_k depending on grid resolution. It is convenient to use λ_k as a way to create wave fields with the same level of nonlinearity, but different levels of non-gaussianity.

We use a very refined 321x71 grid. When $\lambda_k = 0.003$, the number of interactions is relatively small (10^8), and the wave field is nearly gaussian, with very slow evolution. With the increase of λ_k, the number of interactions increases to $7 \cdot 10^9$. We are interested in spectral shape at the advanced stage of the evolution, when the self-similar shape is reached. For small λ_k the shape function is close to the KE spectral shape (figure 10). With the increase of λ_k the shape function approaches a different form resembling Pierson-Moskowitz spectral shape.

VI. Conclusions and implications

Thus, we have demonstrated that although non-gaussianity is weak, in the long term it leads to considerable distortion of the spectral shape. At the same time, integral parameters of a wave field appear to be less affected, with the error remaining within the uncertainty introduced by wave breaking, which the DNS modelling has to take into account. The spectral shape obtained by the DNS appears to be in much closer agreement with observations of mature sea states than the KE spectral shape.

The findings of this project have numerous important implications. First, they are of crucial importance for all applications where the shape of the wave spectrum is significant, rather than just its integrated description, in particular for probability estimates of extreme wave events, design or coastal hazard risk assessments, sediment transport models, etc. Second, it is well known that the wind wave models based on the KE are optimized for certain frequency and directional resolutions against the available measurements. Knowledge of systematic errors in models can drastically improve the quality of such optimizations, and thus improve the quality of wind wave modelling. Third, the findings of this study provide an insight on the role of non-gaussianity in kinetic models, which is significant for a wide context of wave turbulence in various branches of physics.

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Romero, L. & Melville, W.K. 2010 Airborne observations of fetch-limited waves in the Gulf of Tehuantepec. J. Phys. Oceanogr. 40, 441-465.

List of publications/reports from the project with complete references

Annenkov S.Y., Shrira V.I. When is the dynamic non-Gaussianity essential for water wave fields? WISE 2019, Jozankei, Hokkaido, Japan, 12--16 May 2019

Annenkov S.Y., Shrira V.I. Evolution of random wave fields and the role of the statistical closure. 8th International Symposium on Bifurcations and Instabilities in Fluid Dynamics, University of Limerick, Ireland, 2019

Annenkov S.Y., Shrira V.I. Evolution of weakly nonlinear random wave fields: kinetic equations vs the Zakharov equation. IX-th International Conference "SOLITONS, COLLAPSES AND TURBULENCE: Achievements, Developments and Perspectives" (SCT-19) Yaroslavl, Russia 5 - 9 Aug 2019.

Annenkov S., Shrira V., Romero L., Melville W.K., Le Merle E., Hauser D. 2021 Wave development and transformation under strong offshore winds: modelling by DNS and kinetic equations and comparison with airborne measurements. EGU General Assembly Conference Abstracts, EGU21-10437.

Annenkov S., Shrira V., Romero L., Melville W.K. 2021 Long-term evolution of directional spectra of wind waves modelled by DNS and kinetic equations, and comparison with airborne measurements. EGU General Assembly Conference Abstracts, EGU21-10435.

Annenkov, S.Y., Shrira, V.I. 2021 Effects of non-gaussianity on evolution of a random wind wave field. Phys. Rev. E, submitted

Annenkov S.Y., Shrira V.I. Why wind wave modelling fails to capture observed mature wave spectra? The 27th WISE meeting, Bergen, Norway, September 2021

Future plans

(Please let us know of any imminent plans regarding a continuation of this research activity, in particular if they are linked to another/new Special Project.)

After we have demonstrated that the neglect of finite non-gaussianity effects leads to a considerable distortion of the spectral shape, the natural next step is to study the role of the terms describing these effects in wave kinetics. We plan to extend the existing algorithm for the generalised kinetic equation and to create, for the first time, the numerical model of wind waves within the kinetic theory with the account for weak nonlinearity and weak non-gaussianity. We will also perform direct comparisons with the DNS, and examine implications for freak wave predictions. These are some of the most important objectives of the new Special Project (2022-2024)